|  |  |
| --- | --- |
| Probl  # | Pts |
| 1 | 25 |
| 2 | 45 |
| 3 | 40 |
| 4 | 45 |
|  |  |
| Sum | 100% |

GRADE CALCULATION = round (

(pts#1/.25+ pts#2/.45+ pts#3/.40+ pts#4/.45)/4, 0)

[25 pts] Problem #1

Given the (already familiar) language L = {anbmcp: m > n + p > 0}

(i) *Describe* a TM to decide this language

(ii) *Describe* a TM to decide the complement of this language

Answer:

[0 pts] 01. (The assumption is that at the beginning of the procedure the head is at the extreme left).

[2 pts] 02. (Doing the verification w ∈ a\*b+c\* is not essential but it is OK if you do it! Notice that the set of strings of the form a\*b+c\* includes "b" all by itself as a string).

[3 pts] 03. (There will be four cases: (i) the string contains only b’s, that is n = p = 0, (trivial) M *accepts* immediately; (ii) the string contains only a’s and b’s, that is p = 0, (also trivial), M crosses off a “*b*” for each “*a*”, *accept* if there is at least one *b* not crossed off after crossing off all the *a’s*; (iii) the string contains only *b’s* and *c’s*, that is n = 0, (last trivial case), M crosses off a "*b*" for each “*c*”, *accepts* if there is at least one *b* not crossed off after crossing off all *c’s*; (iv) the string what we will call “normal” includes *a’s, b’s* and *c’s*. This is the case that we will be answering below with strings from a+b+c+).

(i)

[5 pts]

M = “On input w:

1. Starting from the extreme left scan the tape (towards the right) and cross off a *b* for each *a* that is being crossed off. Keep zig zagging from left to right crossing off one *b* for each *a*. If there are no more crossed off *a’s* (since there are more *b's* than a's, that is m > n + p), go to the next step.

2. Scan, moving towards the right of the last crossed off *b*. Cross off a *b* for each *c* towards the right on the tape. Zig-zag between *b’s* and *c’s,* crossing off all the *c’s* (since there must be more b’s than c’s); *accept* if after crossing off all the *c’s*, there is at least one *b* not crossed off. Otherwise *reject.*”

(ii)

[10 pts] Language L given above is a *recursive* language since it will halt in all cases and thus it is decidable. Therefore, its complement exists and it is described below.

[5 pts]

Mc = “On input w:

1. Run TM M described above. If M accepts, *reject*. Otherwise, if M rejects, *accept*.”

[45 pts] Problem #2

*Describe* in detail a TM M for the language L = L1 - L2 for the following cases:

(i) L1 and L2 are both *recursive*

(ii) L1 is *r.e.* and L2 is *recursive*

(iii) L1 is *recursive* and L2 is *r.e.*

(iv) L1 and L2 are both *r.e.*

where M1 is the TM that is associated with L1 and M2 is the TM associated with L2.

Answer:

[5 pts] Since L = L1 - L2 = L1 ∩ (L2)c, we need a TM Mc associated with (L2)c.

[5 pts] In class, we were given TM Mc for the case when the language is recursive.

[5 pts] In class, the TM MINT for the intersection was given for both the *recursive* and the *r.e.* cases. Therefore:

[10 pts] (i)

In this case, as stated in class, M1 exists for a recursive language and also Mc exists for a recursive language. Therefore:

M = "On input w:

1. Run M1 on w; if it rejects, *reject.*

2. Run Mc on w, if it accepts, *accept*. Otherwise, *reject.*”

(M accepts w if both M1 and Mc accepts it. If either of them rejects, M rejects too.)

[10 pts] (ii)

In this case, as stated in class, M1 exists for a r.e. language and also Mc exists for a recursive language. Therefore:

M = "On input w:

1. Run M1 on w; if it rejects, *reject.*

2. Run Mc on w, if it accepts, *accept*. Otherwise, *reject.*”

(M accepts w if both M1 and Mc accepts it. If either of them rejects, M rejects too.)

[5 pts] (iii):

In this case, as stated in class, M1 exists for a recursive language but Mc does not exist for a r.e. language. Therefore, there is no solution.

[5 pts] (iv):

In this case, as stated in class, M1 exists for a r.e. language but Mc does not exist for a r.e. language. Therefore, there is no solution.

[40 pts] Problem #3

Run the TM below (execute all the required transitions) under the three inputs: (i) Babaa, (ii) Baa, (iii) Baba. (Here B is the blank symbol).

q0

q1

q4

q2

q3

q5

q6

h

B/B, R

a/B, R

b/B, R

B/B, R

B/B, L

B/B, L

a/B, L

b/B, L

b/b, R

a/a, R

b/b, R

a/a, R

b/b, L

a/a, L

B/B, R

B/B, R

B/B, L

Answer:

[15 pts] (i)

the 1st is 0 pts: q0Babaa → Bq1abaa → BBq2baa → BBbq2aa → BBbaq2a → BBbaaq2 → BBbaq3a →

→ BBbq4aB → BBq4baB → Bq4BbaB → BBq1baB → BBBq5aB → BBBaq5B → BBB q6aB →

+ 2pt for writing the system crashes!

[10 pts] (ii) [2 pts]

q0Baa → Bq1aa → BBq2a → BBaq2 → BBq3a → Bq4BB → BBq1B → BhBB halts.

[15 pts] (iii)

q0Baba → Bq1aba → BBq2ba → BBbq2a → BBbaq2 → BBbq3a → BBq4bB →

[2 pts]

→ Bq4BbB → Bq4BbB → BBq1bB → BBBq5B → BBq6BB → BBBhB halts.

[45pts] Problem #4.

Given a TM by its  function below, compute the final output of the TM under the strings:

[15pts] (i) 0010, and

[10pts] (ii) 0100.

[10pts] (iii) write the TM detailing its 7 components explicitly

[10pts] (iv) draw this TM

Note: the symbol "B" stands for "blank" (the ⊥ symbol in the class notes):

(Hint: you start from q0 followed by 0010, that is: q00010 🡺Bq1010 🡺... and so on.)

|  |  |  |  |
| --- | --- | --- | --- |
| (q0, 0) = (q1, B, R) | (q2, 0) = (q3, 1, L) | (q3, 1) = (q3, 1, L) | (q4, B) = (q6, 0, R) |
| (q0, 1) = (q5, B, R) | (q2, 1) = (q2, 1, R) | (q3, B) = (q0, B, R) | (q5, 0) = (q5, B, R) |
| (q1, 0) = (q1, 0, R) | (q2, B) = (q4, B, L) | (q4, 0) = (q4, 0, L) | (q5, 1) = (q5, B, R) |
| (q1, 1) = (q2, 1, R) | (q3, 0) = (q3, 0, L) | (q4, 1) = (q4, B, L) | (q5, B) = (q6, B, R) |

Answer:

[15pts] (i)

no pts 1st two: q00010 🡺Bq1010 🡺 B0q110 🡺 B01q20 🡺 B0q311 🡺 Bq3011 🡺 q3B011 🡺 Bq0011 🡺 BBq111 🡺 BB1q21 🡺 BB11q2 🡺 BB1q41 🡺 BBq41B 🡺 Bq4BBB 🡺 B0q6BB 🡺

[2pts] for saying “halts at q6 with a 0 on tape”

[10pts] (ii)

q00100 🡺Bq1100 🡺 B1q200 🡺 Bq3110 🡺 q3B110 🡺 Bq0110 🡺 BBq510 🡺

🡺BBBq50 🡺 BBBBq5 🡺 BBBBBq6 [2pts] for saying “system halts at q6 empty tape.”

[10pts] (iii)

TM = (Q, , , d, q0, qaccept, qreject), where

[1pts] Q = {q0, q1, q2, q3, q4, q5, q6)

[1pts]  = {0, 1}

[1pts]  = {0, 1, B}

[2pts] qaccept = halt = q6 = this is the final state (as you can see from the two examples above)

[5pts] qreject = there is no reject state = 

[10pts] (iv) draw this TM

q0

q4

q1

q2

q5

q6

0/B, R

1/B, R

B/B, R

1/1, R

B/B, R

B/B, L

1/B, R

0/B, R

0/0, R

0/1, L

B/0, R

1/1, R

q3

1/B, L

0/0, L

1/1, L

0/0, L